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Effects of magnetic field and suction/injection on convection heat transfer of non-Newtonian power-law fluids past a power-law stretched sheet with surface heat flux

Chien-Hsin Chen *

Department of Mechanical Design Engineering, National Formosa University, Huwei, Yunlin 632, Taiwan

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Abstract

The problem of magneto-hydrodynamic flow and heat transfer of an electrically conducting, non-Newtonian power-law fluid past a stretching sheet in the presence of a transverse magnetic field is analyzed. The surface of the stretching sheet is assumed to move with a power-law velocity and subject to uniform surface heat flux. The effects of suction or injection at the surface are considered. The resulting governing equations are transformed into nonlinear ordinary differential equations using appropriate transformations and then solved numerically based on central-difference approximations. The solution is found to be dependent on five governing parameters including the magnetic field parameter, the power-law fluid index, the sheet velocity exponent, the suction/blowing parameter, and the generalized Prandtl number. A systematical study is carried out to illustrate the effects of these major parameters on the sheet surface temperature, fluid temperature distributions in the boundary layer, the skin-friction coefficient and the local Nusselt number.

Keywords: Magnetic field effects; Non-Newtonian power-law fluid; Power-law stretched sheet; Suction/injection; Surface heat flux

1. Introduction

The study of flow and heat transfer for an electrically conducting fluid past a heated surface has attracted the interest of many investigators in view of its applications in many physical, geophysical and industrial fields. To be more specific, it may be pointed out that many manufacturing processes involve the cooling of continuous sheets or filaments by drawing them through a quiescent fluid. These sheets or filaments are usually stretched during the drawing process. It is known that the properties of the final product depend to a great extent on the rate of cooling. By drawing such sheets in an electrically conducting fluid subject to a magnetic field, the rate of cooling can be controlled and the final product of required characteristics can be obtained. Another important application of MHD flow to metallurgy lies in the purification of molten metals from non-metallic inclusion by the application of magnetic field. In view of these, the problem MHD flow and heat transfer of an electrically conducting fluid past a continuously stretching surface have been studied extensively by many researchers in recent years. Chiam [1] studied the boundary layer flow of a Newtonian fluid over a stretching plate in the presence of a transverse magnetic field. Pop and Na [2] performed an analysis for the MHD flow past a stretching permeable surface. Chandran et al. [3] examined the effect of magnetic field on the flow and heat transfer from a surface moving with a constant velocity. Vajravelu and Hadjinicolaou [4] studied the flow and heat transfer characteristics in an electrically conducting fluid near an isothermal sheet, taking into account the effects of free convection and internal heat generation. Recently, Mukhopadhyay et al. [5] have dealt with a steady two-dimensional flow of an electrically conducting incompressible fluid at a heated stretching sheet. In their analysis the flow is permeated by a uniform transverse magnetic field and the fluid viscosity is assumed to vary as a linear function of temperature. However, the above-mentioned studies are restricted to viscous flows of Newtonian fluids.

* Tel.: +886 5 6315354; fax: +886 5 6363010.
E-mail address: chchen@nfu.edu.tw.

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Nomenclature

B	magnetic field strength
C	constant
C_f	skin-friction coefficient, $\tau_w/(\rho u_w^2/2)$
c_p	specific heat
f	dimensionless stream function
f_w	suction/blowing parameter, $f_w = -\upsilon_w Re_x^{1/(n+1)}/u_w$
h	local heat transfer coefficient
k	thermal conductivity
K	consistency coefficient
M	magnetic field parameter, $\sigma B^2 x/\rho u_w$
n	power-law fluid index
Nu_x	local Nusselt number, hx/k
Pr	generalized Prandtl number, $(xu_w/\alpha)Re_x^{-2/(n+1)}$
p	sheet velocity exponent
q_w	surface heat flux
Re_x	local Reynolds number, $\rho u_w^{2-n} x^n/K$
T	temperature
u	fluid velocity component in x-direction
u_w	sheet velocity
υ	fluid velocity component in y-direction
x	streamwise coordinate
y	cross-stream coordinate

Greek symbols

α	thermal diffusivity
η	similarity variable
ρ	density
σ	electrical conductivity
τ_w	wall shear stress
ϕ	dimensionless temperature
ψ	stream function

Subscripts

w	quantities at wall
∞	quantities at the free stream

Superscript

$'$	differentiation with respective to η

The study of non-Newtonian fluid flow and heat transfer over a stretching surface may gain importance due to numerous industrially important fluids exhibit non-Newtonian fluid behavior such as polymer solutions, molten plastics, pulps, and foods. The flow and heat transfer problems of viscoelastic fluids over a stretching sheet subject to a transverse magnetic field have been considered in the past years by several investigators (see for example, Rajagopal et al. [6], Dandapat and Gupta [7], Datti et al. [8], Siddheshwar and Mahabaleswar [9], and Khan et al. [10]). Also, the MHD flow of a micropolar fluid past a continuously moving plate has been analyzed in the recent studies conducted by Seddeek [11], Eldabe et al. [12], and Rahman and Sattar [13].

For the non-Newtonian power-law fluids, the hydrodynamic problem of the MHD boundary layer flow over a continuously moving surface has been dealt with by several authors (e.g. Andersson et al. [14], Cortell [15], and Mahmoud and Mahmoud [16]). The effect of magnetic field is found to decrease the velocity distribution and thus to increase the skin-friction coefficient. However, relatively less attention has been paid to the accompanying heat transfer problem of power-law fluids past a stretching surface in the presence of a magnetic field. In the present work we investigate the flow and heat transfer of an electrically conducting, non-Newtonian power-law fluid past a continuously stretching sheet under the action of a transverse magnetic field. The sheet is assumed to be subject to a surface heat flux and surface suction or injection. Also, the surface velocity is assumed to vary with the distance from the slot in a power-law form, i.e. $u_w = Cx^p$, where C and p are constants. From the present formulation it is found that the problem under consideration is governed by five parameters, that is, the power-law fluid index n, the magnetic field parameter M, the sheet velocity exponent p, the suction/injection parameter f_w, and the generalized Prandtl number Pr. Representative temperature profiles and the local Nusselt number are presented for various values of the major problem parameters to explore the effects of these parameters on the heat transfer characteristics.

2. Analysis

Let us consider a steady two-dimensional flow of an incompressible, electrically conducting fluid obeying the power-law model past a permeable stretching sheet. The origin is located at the slit through which the sheet is drawn through the fluid medium, the x-axis is chosen along the sheet and y-axis is taken normal to it. This continuous sheet is assumed to move with a velocity according to a power-law form, i.e. $u_w = Cx^p$, and be subject to a surface heat flux. Also, a magnetic field of strength B is applied in the positive y-direction, which produces magnetic effect in the x-direction. The magnetic Reynolds number is assumed to be small so that the induced magnetic field is negligible in comparison to the applied magnetic field. We further assume that there is no applied electric field and the Hall effect is neglected. Moreover, Joule heating and viscous dissipation are neglected in the present analysis. Under the foregoing assumptions and invoking the usual boundary layer approximations, the problem is governed by the following equations:

$$\frac{\partial u}{\partial x} + \frac{\partial \upsilon}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + \upsilon\frac{\partial u}{\partial y} = \frac{K}{\rho}\frac{\partial}{\partial y}\left(\left|\frac{\partial u}{\partial y}\right|^{n-1}\frac{\partial u}{\partial y}\right) - \frac{\sigma B^2 u}{\rho} \tag{2}$$

$$u\frac{\partial T}{\partial x} + \upsilon\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \tag{3}$$

where u and υ are the velocity components, T is the temperature, B is the magnetic field strength, K is the consistency coefficient, n is the flow behavior index, ρ is the density, σ is the electrical conductivity, and α is the thermal diffusivity. The appropriate boundary conditions are given by

$$u_w(x) = Cx^p, \quad \upsilon = \upsilon_w, \quad \partial T/\partial y = -q_w/k \quad \text{at } y = 0,\ x > 0 \tag{4}$$

$$u \to 0, \quad T \to T_\infty \quad \text{as } y \to \infty \tag{5}$$

where υ_w is the surface mass flux and q_w is the surface heat flux. It should be noted that positive p indicates that the surface is accelerated while negative p implies that the surface is decelerated from the slit. Also note that positive υ_w is for fluid injection and negative for fluid suction at the sheet surface.

To proceed, we introduce the dimensionless variables

$$\eta = \left(\frac{C^{2-n}}{K/\rho}\right)^{1/(n+1)} x^{[p(2-n)-1]/(n+1)} y \tag{6}$$

$$\psi = \left(\frac{C^{1-2n}}{K/\rho}\right)^{-1/(n+1)} x^{[p(2-n)+1]/(n+1)} f \tag{7}$$

$$\phi = \frac{(T - T_\infty) Re_x^{1/(n+1)}}{q_w x/k} \tag{8}$$

where the dimensionless stream function f satisfies the continuity equation with $u = \partial\psi/\partial y$ and $\upsilon = -\partial\psi/\partial x$. Under the transformations (6)–(8), the differential equations (2) and (3) reduce to

$$\left(|f''|^{n-1} f''\right)' + \frac{p(2n-1)+1}{n+1} f f'' - p(f')^2 - M f' = 0 \tag{9}$$

$$\frac{1}{Pr}\phi'' + \frac{p(2n-1)+1}{n+1} f\phi' + \frac{p(2n-1)-1}{n+1} f'\phi = 0 \tag{10}$$

with the boundary conditions

$$f'(0) = 1, \qquad f(0) = \frac{n+1}{p(2n-1)+1} f_w$$

$$\phi'(0) = -1 \tag{11}$$

$$f'(\infty) = 0, \qquad \phi(\infty) = 0 \tag{12}$$

where M is magnetic field parameter, f_w is the suction/injection parameter, Pr is the generalized Prandtl number for the power-law fluid, and the primes indicate the differentiations with respective to η. The parameters M, f_w and Pr are defined, respectively, as

$$M = \frac{\sigma B^2 x}{\rho u_w} \tag{13}$$

$$f_w = -\frac{\upsilon_w}{u_w} Re_x^{1/(n+1)} \tag{14}$$

$$Pr = \frac{x u_w}{\alpha} Re_x^{-2/(n+1)} \tag{15}$$

where

$$Re_x = \frac{\rho u_w^{2-n} x^n}{K} \tag{16}$$

is the local Reynolds number. Note here that the magnetic field strength B should be proportional to x to the power $(p-1)/2$ to eliminate the dependence of M on x, i.e. $B(x) = B_0 x^{(p-1)/2}$ where B_0 is a constant. Quantities of main interest include the velocity components u and υ, the skin-friction coefficient, and the local Nusselt number. In terms of the new variables, the velocity components can be expressed as

$$u = u_w f' \tag{17}$$

$$\upsilon = -u_w Re_x^{-1/(n+1)} \left[\frac{p(2n-1)+1}{n+1} f + \frac{p(2-n)-1}{n+1} \eta f'\right] \tag{18}$$

The wall shear stress is given by

$$\tau_w = \left[K\left|\frac{\partial u}{\partial y}\right|^{n-1} \frac{\partial u}{\partial y}\right]_{y=0} = \rho u_w^2 Re_x^{-1/(n+1)} \left|f''(0)\right|^{n-1} f''(0) \tag{19}$$

The skin-friction coefficient is defined as

$$C_f = \frac{\tau_w}{\rho u_w^2/2} = 2Re_x^{-1/(n+1)} \left|f''(0)\right|^{n-1} f''(0)$$

or

$$C_f Re_x^{1/(n+1)} = 2\left|f''(0)\right|^{n-1} f''(0) \tag{20}$$

The local heat transfer coefficient is

$$h = \frac{q_w}{T_w - T_\infty} = \frac{k Re_x^{1/(n+1)}}{x\phi(0)} \tag{21}$$

The local Nusselt number is given by

$$Nu_x = \frac{hx}{k} = \frac{Re_x^{1/(n+1)}}{\phi(0)}$$

or

$$Nu_x Re_x^{-1/(n+1)} = \frac{1}{\phi(0)} \tag{22}$$

3. Results and discussion

The system of transformed governing equations (9) and (10) with the boundary equations (11) and (12) is solved numerically using the central-difference scheme similar to that described in [17]. The transformed equations were first written as a system of first-order nonlinear differential equations, which was then solved by a Newton iteration procedure. In the calculations, a step size of $\eta = 0.01$ were found to be satisfactory in obtaining sufficient accuracy within a tolerance less than 10^{-6} in nearly all cases. To assess the accuracy of the present method, comparisons of the present results to those of previous studies are made for several limiting cases. The flow part of the present problem has been investigated for a linearly stretching sheet ($p = 1$). Andersson et al. [14] explored numerically the effect of magnetic field on the flow characteristics. Liao [18] used the homotopy analysis method (HAM, a powerful innovated analytic method for nonlinear problems, proposed by the same author [19]) to present analytical solutions for n and M in general cases. Table 1 shows the comparison of $-f''(0)$ for various values of n and M with $p = 1$ and $f_w = 0$. As can be seen, the results agree well among these three sets. The heat transfer results have been obtained by Ali [20] and Elbashbeshy [21] for a Newtonian fluid in the absence of the magnetic field, i.e.

Table 1
Comparison of $-f''(0)$ for various values of n and M with $p=1$ and $f_w=0$

n	M	Andersson et al. [14]	Liao [18]	Present study
0.4	0.0	1.273		1.27294
	0.5	1.811		1.81095
	1.0	2.284		2.28377
	1.5	2.719		2.71859
	2.0	3.127		3.12650
0.8	0.0	1.029	1.028	1.02919
	0.5	1.309	1.308	1.30790
	1.0	1.544	1.544	1.54411
	1.5	1.754	1.754	1.75401
	2.0	1.945	1.945	1.94532
	5.0		2.874	2.87438
	10		4.035	4.03451
	50		9.480	9.47935
	100		13.861	13.85856
1.0	0.0	1.000		1.00000
	0.5	1.225		1.22475
	1.0	1.414		1.41421
	1.5	1.581		1.58114
	2.0	1.732		1.73205
1.5	0.0	0.981	0.982	0.98056
	0.5	1.131	1.131	1.13078
	1.0	1.257	1.257	1.25708
	1.5	1.367	1.367	1.36722
	2.0	1.466	1.466	1.46566
	5.0		1.918	1.91827
	10		2.436	2.43597
	50		4.485	4.48487
	100		5.892	5.89207
2.0	0.0	0.980	0.980	0.98000
	0.5	1.093	1.093	1.09286
	1.0	1.187	1.187	1.18730
	1.5	1.269	1.269	1.26921
	2.0	1.342	1.342	1.34198
	5.0		1.670	1.67040
	10		2.033	2.03323
	100		4.234	4.23625

Table 2
Comparison of $1/\phi(0)$ for various values of Pr, p, and f_w with $M=0$ and $n=1$

Pr	p	f_w	Ali [20]	Elbashbeshy [21]	Present study
0.72	−0.2	−0.2	0.62599	0.6309	0.62606
	−0.2	0.6	0.86356	0.8728	0.86352
	1	0.6	0.76235	0.7711	0.76217
1	−0.2	−0.2	0.74053	0.7409	0.74064
	−0.2	0.6	1.11842	1.1180	1.11835
	1	0.6	1.00630	1.0060	1.00616
10	−0.2	−0.2	1.88849	1.8884	1.88847
	−0.2	0.6	7.21160	7.2033	7.21061
	1	0.6	7.09238	7.0921	7.09205

$n=1$ and $M=0$. The heat transfer parameter $1/\phi(0)$ obtained in the present study is compared to those of the previous works [20,21] in Table 2. The results of all these sets are found to be in good agreement. To the author's best knowledge, the heat transfer results are not available in the literature for the uniform surface heat flux case when $n \neq 1$. Table 3 lists the present calculations for the flow and heat transfer characteristics, including the sheet surface temperature $\phi(0)$, the skin-friction coefficient $C_f Re_x^{1/(n+1)}$, and the Nusselt number $Nu_x Re_x^{-1/(n+1)}$ for various values of n, M, and f_w with $Pr=5$ and $p=0.5$. It is apparent from this table that the sheet surface temperature increases with increasing the magnetic field parameter M, but it decreases with increasing the suction/injection parameter f_w. With all other parameter fixed the magnitude of skin-friction coefficient increases with increasing the magnetic field parameter due to the fact that the magnetic field retards the fluid motion and thus increases this coefficient. To impose suction is to increase the skin-friction coefficient, but fluid injection decreases it. Also, the local Nusselt number is decreased as a result of the applied magnetic field. The effect of suction ($f_w > 0$) is found to increase the Nusselt number, whereas injection has the opposite effect. More detailed discussions about the influence of all governing parameters on the local Nusselt number will be presented later in Figs. 6–9.

Table 3
Flow and heat transfer parameters for various values of n, M and f_w with $Pr=5$ and $p=0.5$

n	M	f_w	$\phi(0)$	$C_f Re_x^{1/(n+1)}$	$Nu_x Re_x^{-1/(n+1)}$
0.5	0	−0.2	1.024504	−1.680969	0.976082
		0.0	0.659092	−1.831551	1.517239
		0.6	0.264915	−2.408973	3.774795
	1	−0.2	1.263562	−2.472213	0.791413
		0.0	0.742878	−2.633241	1.346116
		0.6	0.271518	−3.213067	3.683003
	5	−0.2	2.723057	−3.881351	0.367234
		0.0	1.003851	−4.046795	0.996164
		0.6	0.283409	−4.608094	3.528467
1.0	0	−0.2	0.867907	−1.345021	1.152198
		0.0	0.590316	−1.540734	1.694009
		0.6	0.255075	−2.270891	3.920419
	1	−0.2	0.980648	−2.323803	1.019734
		0.0	0.633542	−2.519363	1.578428
		0.6	0.258621	−3.198565	3.866662
	5	−0.2	1.444094	−4.529439	0.692476
		0.0	0.757291	−4.726210	1.320497
		0.6	0.266320	−5.366930	3.754876
1.5	0	−0.2	0.808208	−1.164834	1.237304
		0.0	0.561785	−1.394412	1.780041
		0.6	0.250220	−2.228454	3.996479
	1	−0.2	0.886557	−2.189081	1.127959
		0.0	0.591858	−2.412287	1.689593
		0.6	0.252608	−3.178401	3.958697
	5	−0.2	1.158065	−4.836690	0.863509
		0.0	0.672573	−5.057576	1.486827
		0.6	0.258111	−5.768535	3.874307
1.9	0	−0.2	0.782357	−1.073300	1.278189
		0.0	0.548741	−1.323156	1.822353
		0.6	0.247734	−2.213542	4.036595
	1	−0.2	0.847407	−2.107985	1.180071
		0.0	0.573339	−2.349473	1.744168
		0.6	0.249630	−3.170979	4.005932
	5	−0.2	1.052295	−4.981960	0.950304
		0.0	0.636537	−5.218701	1.571000
		0.6	0.254073	−5.977131	3.935870

Typical temperature distributions are presented at selected values of M for a shear thinning fluid of $n=0.5$ in Fig. 1(a) and for a shear thickening fluid of $n=1.5$ in Fig. 1(b). It is

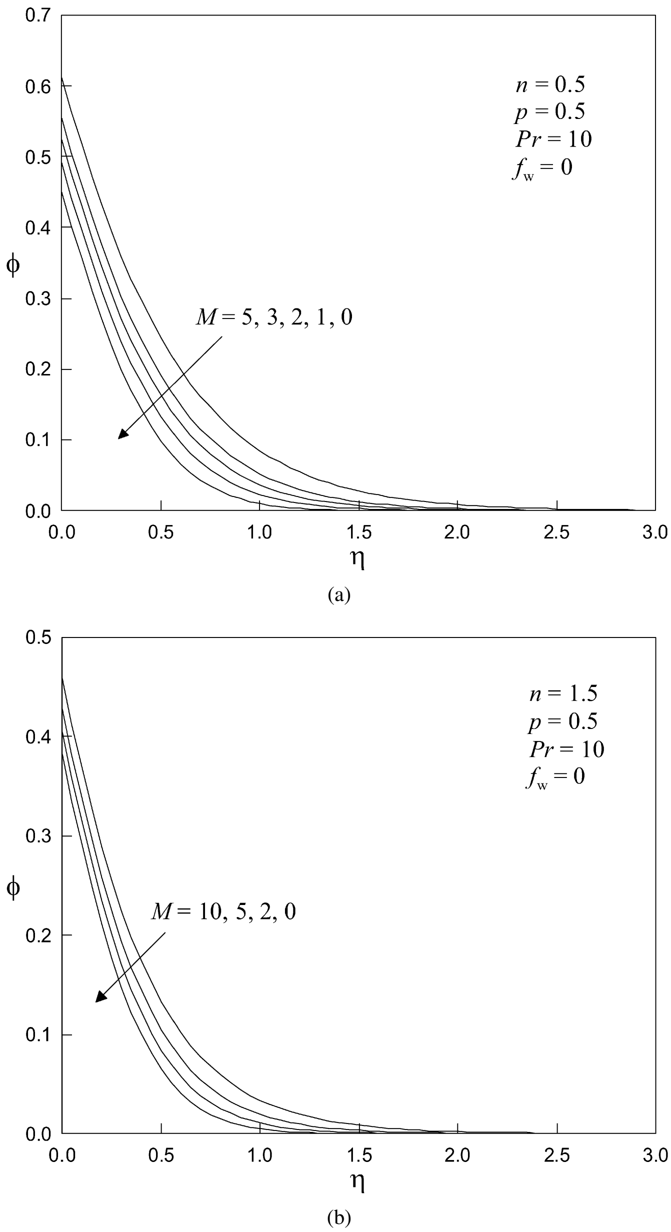


Fig. 1. Temperature profiles for different values of M with $p = 0.5$, $Pr = 10$ and $f_w = 0$; (a) $n = 0.5$; (b) $n = 1.5$.

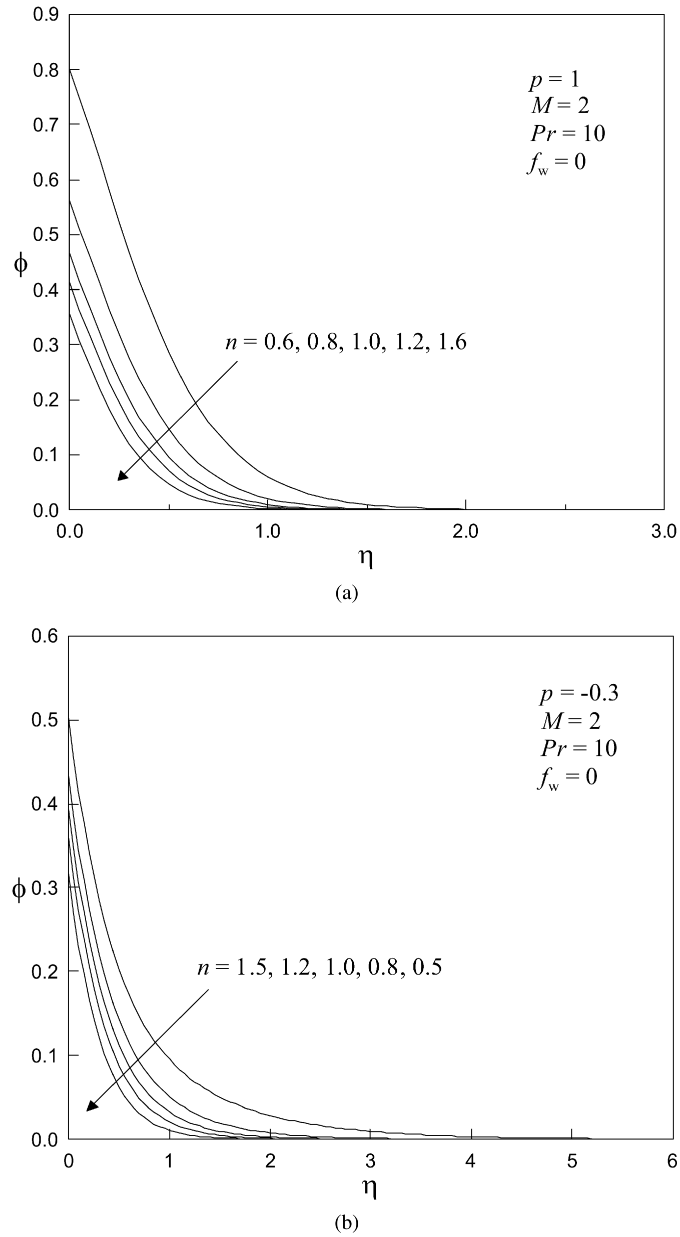
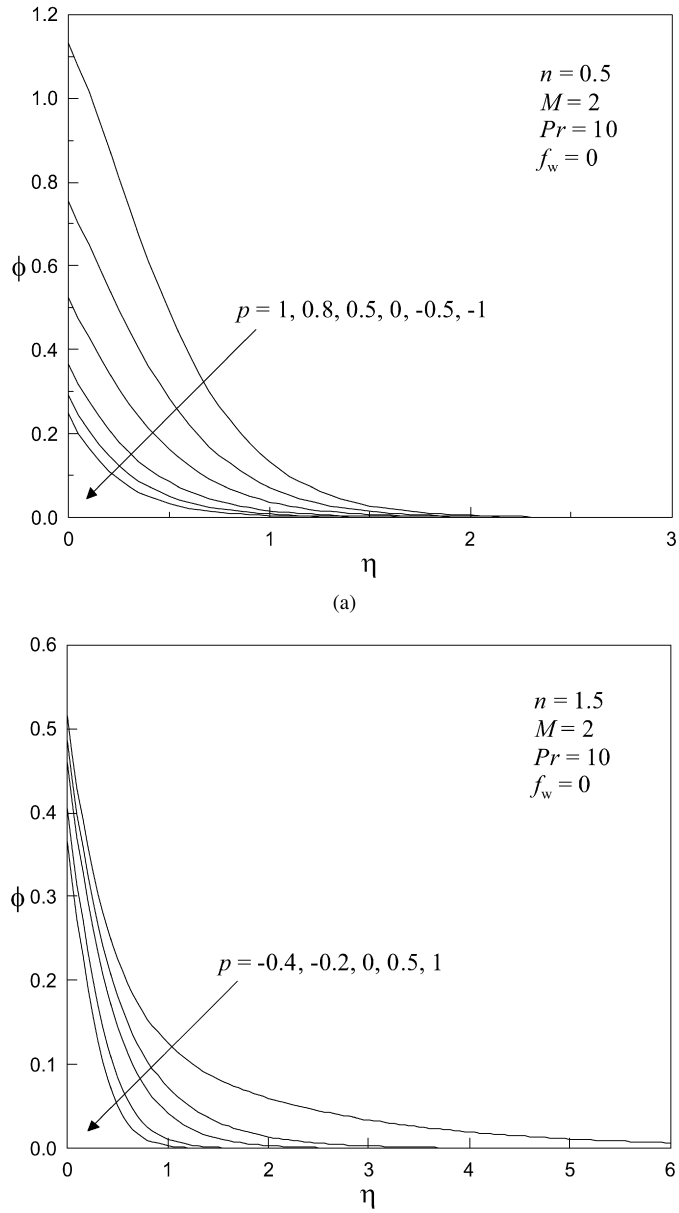


Fig. 2. Temperature profiles for different values of n with $M = 2$, $Pr = 10$ and $f_w = 0$; (a) $p = 1.0$; (b) $p = -0.3$.

clear from these figures that the magnetic field increases the temperature distribution in the boundary layer, and this behavior is more noticeable for the a shear thinning fluid. Also note that the surface temperature and the boundary layer thickness increase with increasing M. Figs. 2(a) and (b) represent the temperature profiles for various values of the power-law index n, respectively, for an accelerated stretching surface ($p = 1$) and for a decelerated stretching surface ($p = -0.3$). It can be seen from Fig. 2(a) that for the accelerated stretching case the fluid temperature decreases as the power-law index increases. On the other hand, Fig. 2(b) shows the opposite phenomenon when the surface is decelerated stretching. Also, the influence of the index n on the wall temperature is more significant when the surface is accelerated stretching. Figs. 3(a) and (b) reveal the influence of sheet velocity exponent p on the temperature distributions for $n = 0.5$ (shear thinning) and $n = 1.5$ (shear thickening), respectively. For a shear thinning fluid, marked increases in the temperature distribution and the surface temperature are caused by increasing the value of p as shown in Fig. 3(a). On the contrary, to increase p is to reduce the temperature for a shear thickening fluid, as indicated in Fig. 3(b). The effects of the suction/injection parameter on the temperature distributions are illustrated in Fig. 4 for a shear thinning fluid with $n = 0.5$. It is obvious that the imposition of the surface injection ($f_w < 0$) broadens the temperature distribution and thus increases the fluid temperature in the boundary layer, whereas the opposite tendency is true for the case of surface suction ($f_w > 0$). Moreover, the surface temperature may be reduced considerably by increasing the suction/injection para-



(a)

(b)

Fig. 3. Temperature profiles for different values of p with $M = 2$, $Pr = 10$ and $f_w = 0$; (a) $n = 0.5$; (b) $n = 1.5$.

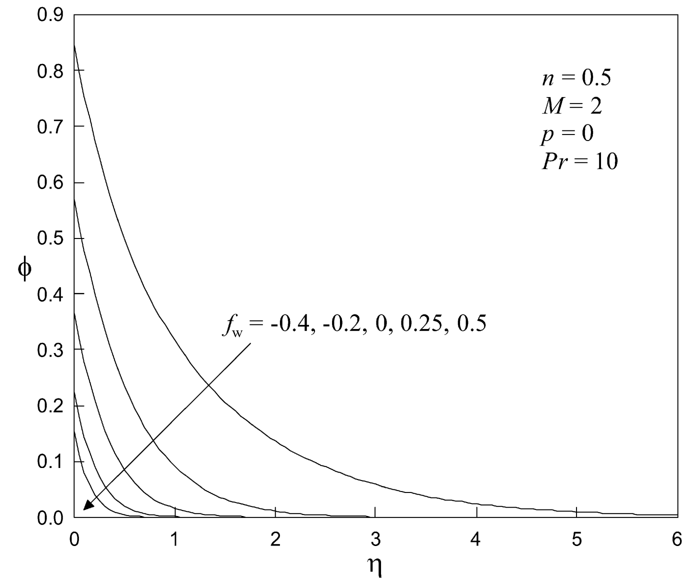


Fig. 4. Temperature profiles for various values of f_w with $n = 0.5$, $M = 2$, $p = 0$ and $Pr = 10$.

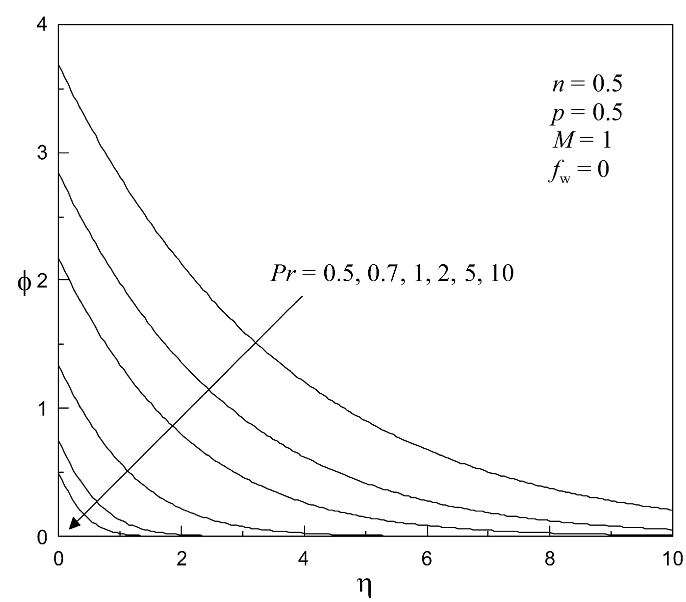


Fig. 5. Temperature profiles for various values of Pr with $n = 0.5$, $p = 0.5$, $M = 1$ and $f_w = 0$.

meter. Fig. 5 reveals the usual effect of the generalized Prandtl number Pr on the temperature profiles for $n = 0.5$, that is, an increase in the Prandtl number will produce a decrease in the thermal boundary layer thickness, associated with the reduction in the temperature profiles. It is noted that the effects of f_w and Pr on the temperature field for shear thickening fluids are similar to those for the shear thinning fluids. The calculations are also performed for a shear thickening fluid of $n = 1.5$ but the results are not presented here for the sake of brevity.

The Nusselt number variation, in terms of $1/\phi(0)$, is presented as a function of the magnetic field parameter M in Fig. 6 for different values of n with $Pr = 10$, $p = 0.5$ and $f_w = 0$. It is evident that the heat transfer parameter $1/\phi(0)$ decreases monotonically with increasing M for a given fluid. Also, the value of Nusselt number becomes higher as n increases for the given parameters. The heat transfer parameter is plotted versus the sheet velocity exponent p in Fig. 7 for several values of the power-law fluid index n. The heat transfer may be enhanced or reduced by increasing p, depending on the fluid index n. From this figure it is observed that the value of $1/\phi(0)$ increases with increasing p for higher values of fluid index (shear thickening), but decreases with p for a lower n (shear thinning fluid). This implies that the wall temperature $\phi(0)$ will increases as p increased for a pseudo-plastic fluid ($n < 1$), but the opposite trend is true for a dilatant fluid ($n > 1$). This phenomenon is consistent with the near-wall behavior of the fluid temperature distribution shown in Fig. 3. It is also interesting to note that the

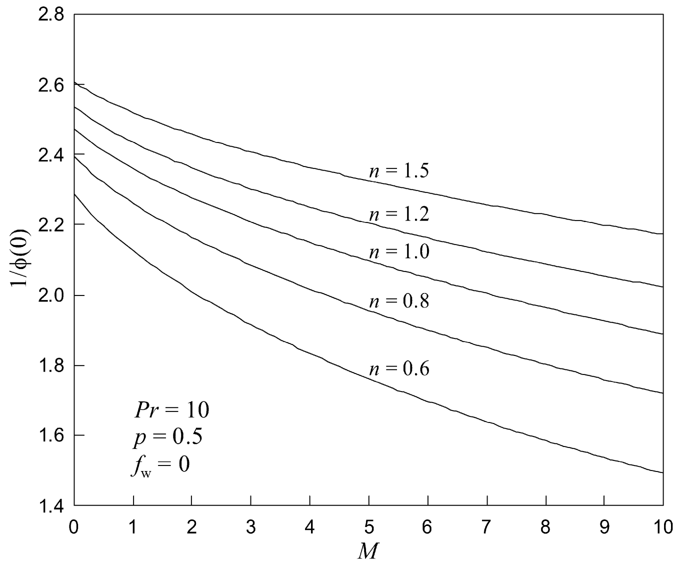


Fig. 6. Variation of $1/\phi(0)$ as a function of M at selected values of n; $Pr = 10$, $p = 0.5$, and $f_w = 0$.

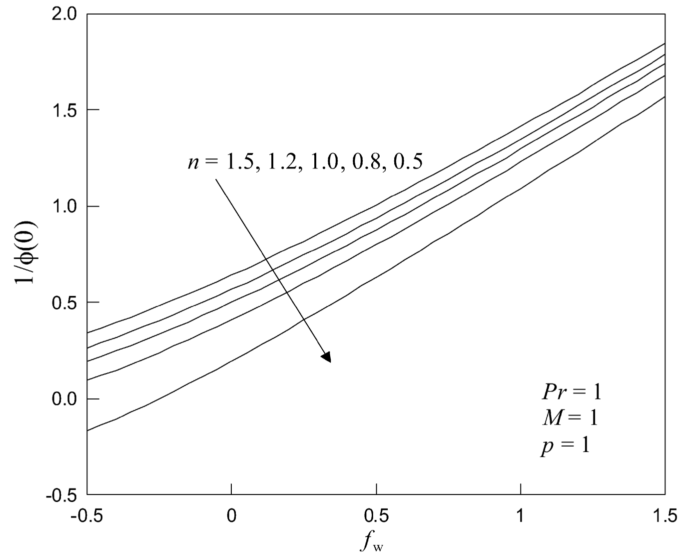


Fig. 8. Variation of $1/\phi(0)$ as a function of f_w for various values of n; $Pr = 1$, $M = 1$, and $p = 1$.

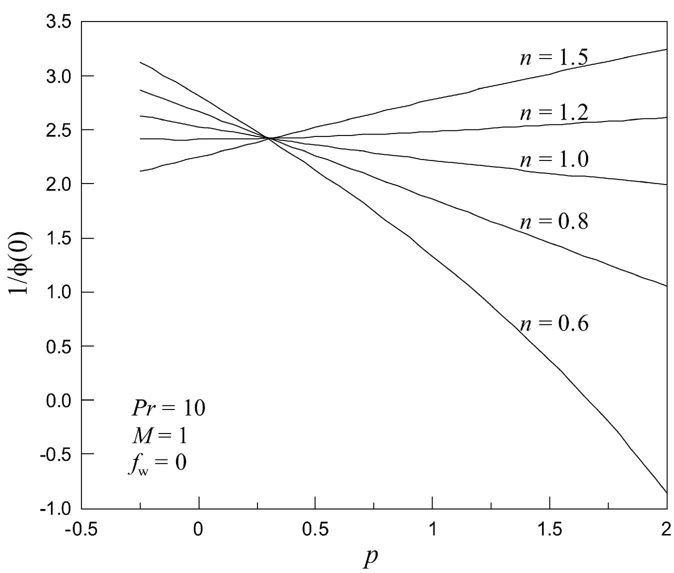


Fig. 7. Variation of $1/\phi(0)$ as a function of p at selected values of n; $Pr = 10$, $M = 1$, and $f_w = 0$.

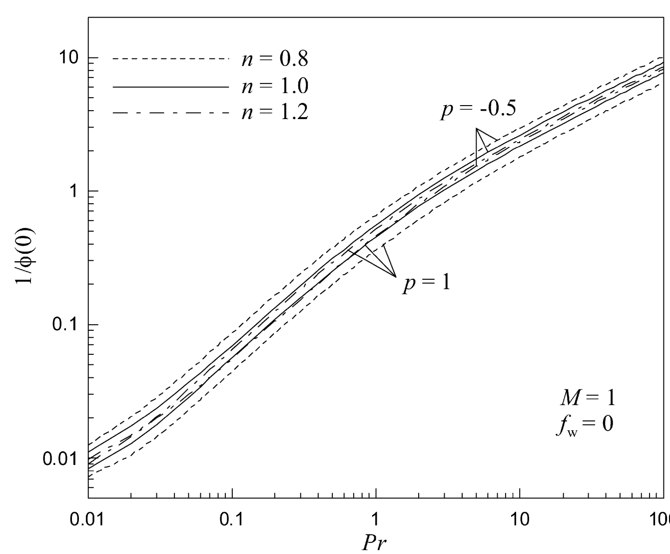


Fig. 9. Variation of $1/\phi(0)$ as a function of Pr for various values of n and p; $M = 1$ and $f_w = 0$.

heat transfer rate increases with n for a larger p; while the trend is reversed for a smaller p, especially when the sheet is decelerated stretching ($p < 0$). The influence of the suction/blowing parameter f_w on the rate of heat transfer from the sheet is illustrated in Fig. 8 at selected values of n. As can be seen, the heat transfer parameter increases with f_w for a specific fluid index. It implies that suction ($f_w > 0$) has the effect to increase the heat transfer rate while injection ($f_w < 0$) decreases it, as compared to an impermeable surface ($f_w = 0$). This phenomenon can be readily understood from the near-wall behavior of the temperature distribution shown in Fig. 4, where the surface temperature decreases as f_w increases, accordingly, the heat transfer parameter $1/\phi(0)$ increases. Fig. 9 reveals the usual effect of generalized Prandtl number on heat transfer for a wide range of Pr, $0.01 \leqslant Pr \leqslant 100$, for various values of n and p. As can be expected, the heat transfer rate increases as Pr increases for given values of n and p. It is also worthy to note here that for a specified Pr the heat transfer rate increases as n increases an accelerated stretching surface ($p = 1$), whereas the opposite trend exists for a decelerated stretching surface ($p = -0.5$).

4. Conclusions

The present work deals with the magnetohydrodynamic flow and heat transfer of a non-Newtonian power-law fluid past a stretching surface, taking into consideration the effects suction or injection at the surface. This continuous surface is assumed to move with a power-law velocity and be maintained at a uniform surface heat flux. The governing equations are transformed into nonlinear ordinary differential equations using appropriate transformations, and then solved numerically by a central-difference approximation. A parametric study is performed to explore the effects of various governing parameters on the fluid flow and heat transfer characteristics. These major parameters include the magnetic field parameter M, power-law fluid index n, sheet velocity exponent p, suction/blowing parameter f_w, and generalized Prandtl number Pr. Numerical results are tabulated for the skin-friction coefficient and the local Nusselt number to reveal the tendency of the solutions. Representative temperature profiles and the heat transfer parameter $1/\phi(0)$ are illustrated for different controlling parameters. It can be drawn from the present results that the magnetic field increases the fluid temperature, raises the sheet surface temperature, and thickens the thermal boundary layer; and these behaviors are more pronounced for the a shear thinning fluid. For shear thinning fluids, noticeable increases in the fluid temperature profile and the sheet surface temperature are produced with an increase in the velocity exponent p. However, shear thickening fluids exhibit the opposite behavior. Furthermore, to increase the suction/injection parameter and the generalized Prandtl number is to reduce the temperatures of the stretching sheet and the fluid, associated with a decrease in the thermal boundary layer thickness.

In general, the magnetic field tends to increase the surface friction but decrease the rate of heat transfer from the sheet. The imposition of suction is to increase both the skin-friction coefficient and the Nusselt number, whereas injection shows the opposite effects. For specific values of Pr, M and f_w, the heat transfer can be either enhanced or reduced, depending on the competition between the effects of n and p. The heat transfer rate increases with n for a larger p, whereas the opposite trend prevails at a small p (in particular, for a decelerated stretching surface, i.e. $p < 0$).

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